Chapter 1

ELECTRONS AND FIELDS

Electron-field interactions play an essential role in the operation of all electron tubes. Fields determine the motion of the electrons in the interelectrode space of a tube, and the electron motion in the interelectrode space determines the currents that flow in the external circuit connected between the electrodes.[1] It is appropriate therefore that we begin this text with a review of the laws that govern the motion of electrons in electric and magnetic fields, as well as some properties of the fields themselves. The discussion of fields in the present chapter will be limited to static electric and magnetic fields. Time-varying fields will be considered in later chapters.

In describing fields and electron-field interactions, we must rely on certain experimental laws of physics. Several such laws from which much of our discussion of the present chapter will develop are:

1. A particle with charge q is acted on by an electric field $\mathbf{E}$ with a force proportional to $q\mathbf{E}$, the force being in the direction of the field if q is positive, and in the opposite direction if q is negative.
2. When a particle with charge q moves with velocity $\mathbf{u}$ in a magnetic field $\mathbf{B}$, it experiences a force proportional to the vector product $q\mathbf{u} \times \mathbf{B}$. The force is in the direction of $\mathbf{u} \times \mathbf{B}$ if the charge is positive, and in the opposite direction if the charge is negative.
3. The electric flux crossing a closed surface surrounding a quantity of charge is proportional to the amount of charge enclosed by the surface and is independent of the shape of the surface. This is known as *Gauss's Law*. A point charge therefore acts as a point source of electric flux, and with each unit of charge there is associated a certain total amount of electric flux.
4. In a static magnetic field the line integral of the magnetic field intensity $\mathbf{H}$ around any closed path surrounding a flow of current I is propor-

[1]See Chapter 6.

tional to the flow of current through a surface enclosed by the path. This relationship is known as *Ampere's Circuital Law.* Lines of magnetic flux close on themselves, and there are no point sources of magnetic flux.

The constants of proportionality used in expressing the foregoing experimental laws, together with the units used to measure mass, length, time, and charge, serve to determine the units in which the electric and magnetic field quantities are measured. Several systems of units are in use at present, each with its own particular advantages. However, the meter-kilogram-second system is perhaps the most widely accepted in electron-tube work, and we shall adhere to it throughout this text. Appendix I lists the mks units in which electric and magnetic field quantities are measured, together with their dimensions. Appendix II lists values of a number of physical constants, and Appendix III presents a summary of relationships governing static electric and magnetic field quantities.

Relativistic effects will be neglected throughout this text; that is, electron velocities will be considered small compared with the velocity of light.

1.1 Electron Motion in an Electric Field

(a) *Change of Kinetic Energy and the Concept of Electric Potential*

A charge of q coulombs in an electric field **E** volts/meter is acted on by a force $q\mathbf{E}$ newtons. The force is in the direction of the field if q is a positive charge, and in the opposite direction for a negative charge. Thus, when an electron moves in an electric field **E**, it experiences a force $-e\mathbf{E}$ newtons, where $-e$ is the charge on the electron, e being equal to 1.602×10^{-19} coulomb. The resulting motion of the electron is described in rectangular coordinates by the three equations,

$$m\frac{d^2x}{dt^2} = -eE_x, \qquad m\frac{d^2y}{dt^2} = -eE_y, \qquad m\frac{d^2z}{dt^2} = -eE_z \qquad (1.1\text{-}1)$$

where m is the mass of the electron, and E_x, E_y, and E_z are the components of **E** in the coordinate directions. If the first of these equations is multiplied by dx on both sides, we obtain

$$m\left[\frac{d(dx/dt)}{dt}\right]dx = m\frac{dx}{dt}d\left[\frac{dx}{dt}\right] = d\left[\tfrac{1}{2}m\left[\frac{dx}{dt}\right]^2\right] = -eE_x dx \qquad (1.1\text{-}2)$$

The right-hand part of this equation states that the portion of the electron's kinetic energy associated with its motion in the x direction is changed by an amount $-eE_x dx$ when the electron moves a distance dx in the x direction under the influence of the field. Similar expressions hold for motion in the y and z directions. It follows, therefore, that if the electron moves a distance

$d\mathbf{l}$ under the influence of the electric field, its net change in kinetic energy is equal to the vector product $-e\mathbf{E}\cdot d\mathbf{l}$. This quantity may be positive or negative depending on the angle between $\mathbf{E}$ and $d\mathbf{l}$.

If the electron travels from point A to point B under the influence of the electric field, its total change in kinetic energy is given by

$$\text{change in k.e.} = -e\int_A^B \mathbf{E}\cdot d\mathbf{l} \tag{1.1-3}$$

where the integral is taken over the path followed by the electron from A to B. This expression is of much importance in determining the behavior of charged particles in electric fields. It holds for time varying fields as well as for static fields.

If the field is constant with time and if the work done by the field on the electron serves only to change the kinetic energy of the electron, the field is said to be conservative. For such a field the integral in Equation (1.1-3) is independent of the path taken from A to B, and we can write

$$-e\oint_{\substack{\text{closed}\\\text{path}}} \mathbf{E}\cdot d\mathbf{l} = 0 \tag{1.1-4}$$

where the integral is taken around a closed path. In this case we can ascribe to each point in space a scalar potential such that the difference in potential between two points is equal to the line integral of $\mathbf{E}$ along any path between them. A potential difference of 1 volt exists between points A and B if the line integral of $\mathbf{E}$ along any path between them is equal to 1 volt. (Potential difference is sometimes called electromotive force or emf.)

If dl is an increment of distance in the direction of an electric field E, the change in potential dV over the distance dl can be expressed as $|dV| = Edl$, and we can write that

$$\mathbf{E} = -\nabla V \tag{1.1-5}$$

where V is the scalar potential. The minus sign implies that the field is directed from regions of higher potential to ones of lower potential. Equation (1.1-5) is valid in regions in which there is space charge as well as regions that are free of charge. From the equation, it is evident that E has the dimensions of volts per meter.

If an electron starts from rest and is accelerated through a potential rise of V volts, it acquires an amount of kinetic energy given by

$$\tfrac{1}{2}mu^2 = -e\int \mathbf{E}\cdot d\mathbf{l} = eV \text{ joules} \tag{1.1-6}$$

Substituting the experimentally measured values for e and m in this, we find the velocity of the electron to be

$$u = 5.93 \times 10^5\sqrt{V} \text{ meters/sec} \tag{1.1-7}$$

A unit of energy frequently used to measure energies gained or lost by an electron is the electron volt. It is equal to 1.602×10^{-19} joule and is the kinetic energy gained by an electron when it is accelerated through a potential rise of 1 volt. If the electron travels between points differing in potential by V volts, its change in kinetic energy is V electron volts.

(*b*) *Electron Trajectories in an Electric Field*

Figure 1.1-1(a) shows two electrodes, A and B, of arbitrary shape. Electrode A is grounded, and electrode B is held at a positive potential with respect to ground. The path that might be followed by an electron which

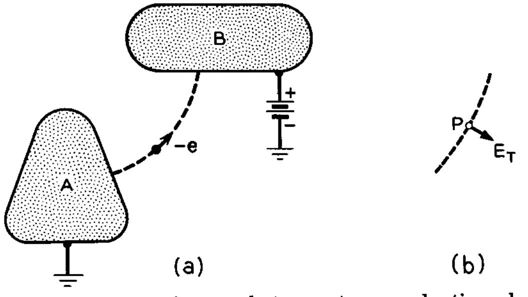

FIG 1.1-1 An electron trajectory between two conducting electrodes.

starts from rest at electrode A and is accelerated in the direction of electrode B is shown by a broken line. Figure 1.1-1(b) shows a curved portion of the path passing through point P. The electric field **E** acting on the electron at point P can be resolved into two components, one parallel to the trajectory and one transverse to it. The transverse component, E_T, is responsible for bending the path of the electron and hence must lie in the plane of curvature of the trajectory. If u is the velocity of the electron at point P and r is the radius of curvature of the trajectory at that point,

$$\frac{mu^2}{r} = eE_T \tag{1.1-8}$$

Since the electron started from rest at electrode A, its kinetic energy at point P is given by

$$\tfrac{1}{2}mu^2 = eV \tag{1.1-9}$$

where V is the potential at point P. Combining these two equations, we obtain

$$r = \frac{2V}{E_T} \tag{1.1-10}$$

Now V and E_T are directly proportional to the voltage applied to electrode B. Since r is equal to twice the ratio of these quantities, it follows that r is independent of the voltage applied to electrode B. Consequently, if the electron starts from rest, its trajectory is the same for all positive voltages applied to electrode B.

A second point, which may seem intuitively clear, follows from similar reasoning. When the linear dimensions in Figure 1.1-1(a) are scaled by a constant factor, the trajectory followed by the electron is scaled by the same factor. Let us suppose that all linear dimensions are multiplied by the factor k and that the voltage applied to electrode B remains unchanged. In this case the potential V at corresponding points between the electrodes will be unchanged. The direction of the electric field intensity also will be unchanged, but its magnitude will be $1/k$ times as great. From Equation (1.1-10), it follows that r becomes k times its previous value, so that r and the trajectory scale with the other linear dimensions.

A third conclusion we can draw from Equation (1.1-10) is that the trajectory is independent of the mass or charge of the particle, provided, of course, that the charge is finite and negative and the mass is not zero. Hence a negative ion would follow the same path as the electron, provided both started from rest at the same point on electrode A.

1.2 Motion in Combined Electric and Magnetic Fields

When a particle with charge q coulombs moves with velocity $\mathbf{u}$ meters per second in a magnetic field $\mathbf{B}$ webers per square meter, it experiences a force $q\mathbf{u} \times \mathbf{B}$ newtons. Thus, an electron moving in a magnetic field $\mathbf{B}$ experiences a force $-e\mathbf{u} \times \mathbf{B}$ newtons, and the resulting acceleration of the electron is $-(e/m)\mathbf{u} \times \mathbf{B}$ meters per second².

The vector $\mathbf{u} \times \mathbf{B}$ has the components $B_z u_y - B_y u_z$ in the x direction, $B_x u_z - B_z u_x$ in the y direction, and $B_y u_x - B_x u_y$ in the z direction, where u_x, u_y, and u_z are the components of $\mathbf{u}$ in the coordinate directions, and B_x, B_y, and B_z are the components of $\mathbf{B}$ in the coordinate directions. If both an electric field and a magnetic field act on an electron, the differential equations describing the motion of the electron are

$$\frac{d^2x}{dt^2} = -\frac{e}{m}\left(E_x + B_z\frac{dy}{dt} - B_y\frac{dz}{dt}\right) \qquad (1.2\text{-}1)$$

$$\frac{d^2y}{dt^2} = -\frac{e}{m}\left(E_y + B_x\frac{dz}{dt} - B_z\frac{dx}{dt}\right) \qquad (1.2\text{-}2)$$

and

$$\frac{d^2z}{dt^2} = -\frac{e}{m}\left(E_z + B_y\frac{dx}{dt} - B_x\frac{dy}{dt}\right) \qquad (1.2\text{-}3)$$

where E_x, E_y, and E_z are the components of the electric field in the coordinate directions. In cylindrical coordinates these equations become

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{e}{m}\left[E_r + B_z r\frac{d\theta}{dt} - B_\theta\frac{dz}{dt}\right] \quad (1.2\text{-}4)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = -\frac{e}{m}\left[E_\theta + B_r\frac{dz}{dt} - B_z\frac{dr}{dt}\right] \quad (1.2\text{-}5)$$

$$\frac{d^2z}{dt^2} = -\frac{e}{m}\left[E_z + B_\theta\frac{dr}{dt} - B_r r\frac{d\theta}{dt}\right] \quad (1.2\text{-}6)$$

We shall find a number of occasions to make use of these equations in later chapters.

Because the force resulting from the magnetic field is perpendicular to the motion of the electron, any component of force parallel to the trajectory must result from the electric field. However, it is the force parallel to the trajectory which changes the electron's kinetic energy, and consequently *only an electric field can change the kinetic energy of an electron.*

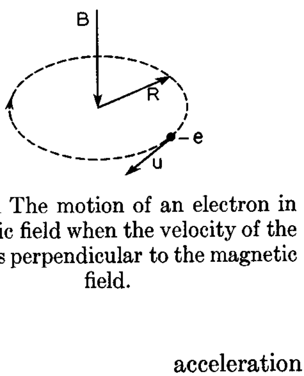

FIG. 1.2-1 The motion of an electron in a magnetic field when the velocity of the electron is perpendicular to the magnetic field.

If the electric field is zero and if the velocity of the electron is perpendicular to the magnetic field, the electron moves in a circular path as illustrated in Figure 1.2-1. The radius R of the path is determined by the relation

$$\text{acceleration} = \frac{u^2}{R} = \frac{e}{m}uB \quad (1.2\text{-}7)$$

or

$$R = \frac{mu}{eB} \quad (1.2\text{-}8)$$

The angular frequency of the circular motion of the electron is given by

$$\omega = \frac{u}{R} = \frac{eB}{m} \quad (1.2\text{-}9)$$

As a simple example of motion in combined electric and magnetic fields, let us consider the case illustrated in Figure 1.2-2. Here, an electric field E lies parallel to the $-y$ direction of a rectangular coordinate system, and a magnetic field B lies parallel to the $-z$ direction. We shall assume that an electron starts from the origin at time $t = 0$ with zero velocity. The electron is initially acted on only by the electric field, but as it advances in the y direction and gains velocity, it is acted on by the magnetic field with a force proportional to the product of its velocity and the magnetic flux density.

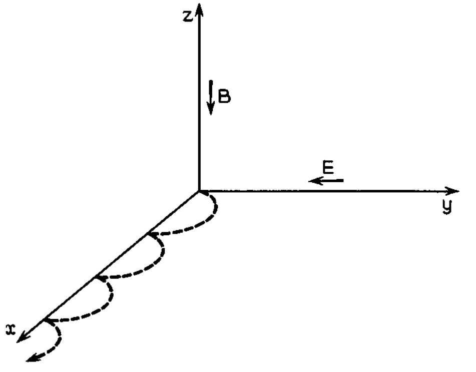

FIG. 1.2-2 The trajectory of an electron which starts from rest in crossed electric and magnetic fields.

As a result, the trajectory is bent back toward the x axis. For this problem Equations (1.2-1), (1.2-2), and (1.2-3) reduce to the two simple equations,

$$\frac{d^2x}{dt^2} = \frac{e}{m}B\frac{dy}{dt}$$
$$\frac{d^2y}{dt^2} = \frac{e}{m}E - \frac{e}{m}B\frac{dx}{dt} \tag{1.2-10}$$

It is easily shown that these equations have the solutions

$$\frac{dx}{dt} = \frac{E}{B}(1 - \cos \omega t)$$
$$\frac{dy}{dt} = \frac{E}{B}\sin \omega t \tag{1.2-11}$$

and

$$x = \frac{E}{\omega B}(\omega t - \sin \omega t)$$
$$y = \frac{E}{\omega B}(1 - \cos \omega t) \tag{1.2-12}$$

where $\omega = eB/m$. Equations (1.2-12) are the equations of a cycloid, the electron trajectory being as illustrated in Figure (1.2-2). Each $2\pi/\omega$ seconds the electron returns to the x axis and then repeats the curved part of the trajectory.

Next let us consider the scaling of electron trajectories in a region in which there is both an electric field and a magnetic field. It is convenient to rewrite Equation (1.1-8) to express the radius of curvature of the trajectory as

$$r = \frac{mu^2}{\text{transverse force}} \tag{1.2-13}$$

where the transverse force in this case may result from both an electric field and a magnetic field. The transverse force, of course, lies in the plane of curvature of the trajectory. Clearly, if we change the electric field intensity and the magnetic flux density in such a manner that the right-hand side of this equation is unchanged for all points on the trajectory, the shape of the trajectory will not be changed. Suppose the electric field intensity at all points is increased by the factor a^2 and the magnetic flux density is increased by the factor a. Then an electron which starts from rest at the beginning of the trajectory and travels to point P on the trajectory will have a^2 times as much energy at point P, and its velocity will be a times as great. The part of the transverse force resulting from the electric field will also be a^2 times as great; and since the part of the transverse force that results from the magnetic field is proportional to the product of u and B, this also will be increased by the factor a^2. Hence both the numerator and denominator of the right-hand side of Equation (1.2-13) will be increased by the factor a^2, and the radius r will be unchanged. Thus, if we increase the electric field intensity at all points in space by the factor a^2 and the magnetic flux density by the factor a, the trajectory of an electron which leaves a given point in space with zero initial velocity will remain unchanged, but the electron will travel a times as fast. (The reader will readily verify this to be the case for the trajectories given by Equations (1.2-12).)

By similar reasoning it is easily shown that, if the linear dimensions of the electrodes are increased by the factor b, and if all the voltages applied to the electrodes are increased by the factor b^2, and if the magnetic flux density *at corresponding points* between the electrodes is unchanged, the electron trajectory will also scale with the other linear dimensions of the system. In this case the electron velocity at corresponding points of the trajectory will be increased by the factor b.

As a final point, we should note that the motion of an electron in an electric or magnetic field is governed entirely by the forces acting on it. The only way we can change the kinetic energy of an electron is to cause the electron to be acted on by an electric field. *Changing the potential in the region does not in itself change the kinetic energy of the electron.*

1.3 Conservation of Energy and Charge

One of the most important laws governing the behavior of physical processes is the principle of conservation of energy. It states that energy can never be created or destroyed. As applied to electron tubes, it tells us that whenever an electron gains kinetic energy, we can in principle account for the source of kinetic energy and show that the source lost an equal amount of energy. Similarly, when an electron loses kinetic energy, we can in principle find an amount of energy which has appeared elsewhere in the system equal to the lost kinetic energy.

Another significant law we learn from experimental physics is the principle of conservation of charge. This principle states that the total charge of a system, both positive and negative, can be changed only by adding charge to the system or removing charge from the system. In later chapters we shall frequently have occasion to consider volume charge densities or "space-charge densities" arising from a large number of electrons in a region of space. If $\rho(x,y,z)$ is the volume charge density, the total charge in an element of volume Δv is $\rho(x,y,z)\,\Delta v$. The principle of conservation of charge tells us that, if this quantity is changing with time, charge is flowing across the surface of the volume element, such that the total amount of charge both inside and outside is constant. Expressed mathematically, the principle states that

$$\int_{\substack{\text{closed}\\\text{surface}}} \mathbf{J}\cdot\mathbf{n}dS = -\frac{\partial}{\partial t}\int_{\text{volume}} \rho(x,y,z)dv \qquad (1.3\text{-}1)$$

where $\mathbf{J}(x,y,z)$ is the current density associated with the flow of charge, and $\mathbf{n}$ is a unit vector normal to the surface element dS and pointing outward. Dividing both sides by Δv and taking the limit as $\Delta v \rightarrow 0$, the left-hand side becomes the divergence of $\mathbf{J}$, and we obtain

$$\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t} \qquad (1.3\text{-}2)$$

This is known as the *equation of continuity*. We shall find a number of occasions to make use of it in later chapters.

1.4 Static Electric Fields — Gauss's Law, Poisson's and Laplace's Equations

(a) *Gauss's Law*

In mks units the electric flux density $\mathbf{D}$ is related to the electric field intensity $\mathbf{E}$ by $\mathbf{D} = \varepsilon\varepsilon_o\mathbf{E}$, where ε is the relative dielectric constant of the medium, and ε_o is the permittivity of free space. The relative dielectric constant ε is a dimensionless constant, which in free space has the value 1.

The constant ε_o is approximately equal to[2] 8.854×10^{-12} and has the dimensions of farads per meter or coulombs per volt per meter. Since **E** has the dimensions of volts per meter, the vector **D** has the dimensions of coulombs per square meter. (The vector **D** is sometimes called the displacement vector.)

If we surround a quantity of charge by a closed surface, a certain total amount of electric flux crosses the surface because of the charge inside. Gauss's Law states that no matter what surface we choose to surround the charge, the total flux crossing the surface is the same. Furthermore, the amount of flux crossing the surface is proportional to the charge enclosed. *Hence with each unit of charge there is associated a certain total amount of flux.* In mks units the flux crossing the surface is numerically equal to the charge in coulombs enclosed by the surface. Gauss's Law therefore can be expressed as

$$\int_{\substack{\text{closed}\\ \text{surface}}} \mathbf{D}\cdot\mathbf{n}dS = \int_{\substack{\text{closed}\\ \text{surface}}} \varepsilon\varepsilon_o\mathbf{E}\cdot\mathbf{n}dS = q \tag{1.4-1}$$

where **n** is a unit vector normal to the surface element dS, and q is the charge enclosed by the surface. If there is a distribution of charge within the region, the theorem can be written in the form

$$\int_{\substack{\text{closed}\\ \text{surface}}} \mathbf{D}\cdot\mathbf{n}dS = \int_{\text{volume}} \rho(x,y,z)dv \tag{1.4-2}$$

where $\rho(x,y,z)$ is the volume charge density, and the integral on the right is taken over the volume enclosed by the surface. Equations (1.4-1) and (1.4-2) are valid even if the surface over which the integrals are taken passes through a conductor or other solid matter, or if it passes through a region of space charge. (However, if the surface element dS lies in a conductor, $\mathbf{E} = 0$, and the flux crossing dS is zero.)

If the volume enclosed by the surface in Equation (1.4-2) is Δv, and if both sides of the equation are divided by Δv, and the limit is taken as $\Delta v \to 0$, we obtain

$$\nabla\cdot\mathbf{D} = \rho \tag{1.4-3}$$

This provides another useful expression of Gauss's Law.

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[2]In mks units the magnetic permeability of free space μ_o is defined to be equal to $4\pi \times 10^{-7}$, and the constants μ_o and ε_o are related by $\mu_o\varepsilon_o = 1/c^2$, where c is the velocity of light. Hence ε_o can be determined by experimental measurement of the velocity of light. It is found that $c \approx 2.996 \times 10^8$ meters/sec, so that $\varepsilon_o \approx 8.854 \times 10^{-12}$, or approximately $1/(36\pi \times 10^9)$.

If q in Equation (1.4-1) is positive, the net electric flux crossing the surface is directed outward, and if q is negative, the net electric flux is directed inward. If the charge enclosed by the surface consists of two equal but opposite charges, the net electric flux crossing the surface is zero.

Two results that follow directly from Gauss's Law and symmetry arguments are:

1. The electric field in free space at a distance r from a point charge q is given by

$$E = \frac{q}{4\pi\varepsilon_o r^2} \text{volts/meter} \tag{1.4-4}$$

2. The electric field in free space outside a cylindrical charge distribution of uniform axial charge density is given by

$$E = \frac{\tau}{2\pi\varepsilon_o r} \text{volts/meter} \tag{1.4-5}$$

where τ is the axial linear charge density in coulombs per meter, and r is the radius at which E is determined.

The concept of lines of electric flux, or field lines, is useful in presenting a picture of an electric field distribution. In the case of two equal but opposite point charges, the electric field lines terminate on the two charges and extend from one charge to the other, the lines being directed from the positive charge to the negative charge. The total number of lines is proportional to the amount of charge at the ends of the field lines. The field lines are parallel to the direction of the electric field, and the number of lines crossing unit area normal to the direction of the field is proportional to the average electric flux density over the unit of area.

Static electric fields are always associated with coulomb charges — either point charges, surface charges, volume charges, or perhaps a combination of the three. In electron-tube work a density of electrons in the interelectrode space of a tube can often be considered to be a volume charge density, or "space-charge density," even though it is really a cloud of individual point charges.

If a point charge is brought close to a conductor, currents flow in the conductor until a charge distribution is built up on its surface which exactly cancels the electric field that would otherwise be present within the conductor. The surface charge is said to be an induced charge. Thus, when electrons are present in the interelectrode space of a vacuum tube, an amount of positive charge equal to the total charge on the electrons is induced on the electrodes or other nearby surfaces, and one can imagine electric field lines extending from the induced surface charges to the electrons in the interelectrode space.

Charges on conductors are always surface charges. A net volume charge density within a conductor would lead to electric fields within the conductor with the result that currents would flow causing neutralization of the charge. Similarly, a static electric field at the surface of a conductor is always normal to the surface of the conductor, since otherwise it would have a component parallel to the surface, and charge would flow along the surface.

By a further application of Gauss's Law, it is easily shown that the electric field intensity E in free space at the surface of the charged conductor is given by

$$E = \frac{\sigma}{\varepsilon_o} \tag{1.4-6}$$

where σ is the surface charge density.

Equation (1.4-6) can be used to obtain an expression for the capacitance of a parallel-plate capacitor. When the capacitor is charged, electric field lines extend from the surface charge on one plate to the surface charge on the other, the charge on the plate at higher potential being positive, and that on the plate at lower potential being negative. If the spacing between plates is small compared with their linear dimensions so that edge effects are negligible, the potential difference from one plate to the other can be expressed as $V = Ed = \sigma d/\varepsilon_o = qd/\varepsilon_o A$, where d is the spacing between the plates, A is the area of a single plate, σ is the surface charge density, and q is the total charge on a single plate. The capacitance of the device is defined as the ratio of q to V, or

$$C = \frac{q}{V} = \frac{\varepsilon_o A}{d} \tag{1.4-7}$$

In mks units, C is measured in farads. If the space between the plates were filled with a material of relative dielectric constant ε, it is easily shown that $E = \sigma/\varepsilon\varepsilon_o$, and $C = \varepsilon\varepsilon_o A/d$.

(*b*) *Poisson's and Laplace's Equations*

Equation (1.4-3) can be written in the form

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\varepsilon\varepsilon_o \mathbf{E}) = \rho \tag{1.4-8}$$

Now $\mathbf{E} = -\nabla V$, and in free space $\varepsilon = 1$. It follows that in a region of free space in which there is a distributed charge density $\rho(x,y,z)$, the potential V is described by the equation

$$\nabla \cdot (\nabla V) = \nabla^2 V = -\frac{\rho}{\varepsilon_o} \tag{1.4-9}$$

This relationship is known as Poisson's Equation.

If there is no space charge in the region, $\rho = 0$, and the potential satisfies Laplace's Equation,

$$\nabla^2 V = 0 \tag{1.4-10}$$

As an example of a problem that can be solved with the aid of Poisson's Equation, let us consider the potential within a long conducting cylindrical tube filled with a uniform charge density ρ_o. (We can imagine that an electron beam of uniform charge density is directed down inside the tube and that the beam just fills the tube.) Using cylindrical coordinates, Poisson's Equation for this problem becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = -\frac{\rho_o}{\varepsilon_o} \tag{1.4-11}$$

since there is no variation of V in the θ or z directions. The reader will readily verify that $V = -(\rho_o/4\varepsilon_o)r^2 + c_1 \ln r + c_2$ is a solution of this equation, where c_1 and c_2 are constants. Evidently $c_1 = 0$, since V is finite at $r = 0$. If the inside radius of the conducting tube is R meters, and if the tube is at zero potential, the potential at radius r is given by $V = (\rho_o/4\varepsilon_o)(R^2 - r^2)$ for $r \leq R$. Positive space charge raises the potential within the cylinder, and negative space charge lowers it.

A problem that can be solved with the aid of Laplace's Equation is that of finding the potential in the region between two long concentric conducting cylinders which are held at different potentials. Since V does not change in the θ or z directions, Laplace's Equation for this problem becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \tag{1.4-12}$$

This equation has the solution $V = c_1 \ln r + c_2$, where c_1 and c_2 are constants. If the inner cylinder is held at potential V_o and the outer cylinder is at zero potential, and if their radii are a meters and b meters, respectively, it is easily shown that $V = (V_o \ln r/b)/(\ln a/b)$. A solution of Laplace's Equation which satisfies a particular set of boundary conditions is always unique, and the first and second derivatives of such a solution are continuous at all points between the bounding surfaces.

Potential distributions can also be obtained by integrating known electric field distributions along the direction of the field. In this case use is made of the relation $\mathbf{E} = -\nabla V$. Thus, if the axial charge density on the inner cylinder in the above problem were specified, we could integrate Equation (1.4-5) with respect to r to obtain the potential as a function of r. In a similar manner, Equation (1.4-4) can be integrated with respect to r to obtain the potential due to an isolated point charge. Thus

$$V = \frac{q}{4\pi\varepsilon_o r} + c_1 \tag{1.4-13}$$

where c_1 is a constant, and r is the distance from the charge q to the point at which V is determined. If V is assumed to be zero at large distances from the point charge, then $c_1 = 0$.

(*c*) *Superposition*

Because Laplace's Equation is linear, the sum of the potentials arising from two or more point charges also satisfies it. If a region of space contains a number of point charges as well as surface charges and volume charges, the potential at point P can be expressed as

$$V_p = \sum \frac{dq}{4\pi\varepsilon_o r} \tag{1.4-14}$$

where dq is a point charge or element of surface charge or volume charge, and r is the distance from the point charge or element of charge to point P.

A problem that can be solved with the aid of Equation (1.4-14) is that of finding the potential at point P outside a conducting sphere with uniform charge density σ. We shall assume that there are no other point charges,

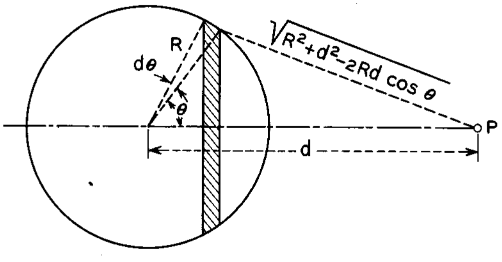


FIG. 1.4-1 A construction which may be used in determining the potential at a point d meters from the center of a uniformly charged conducting sphere.

volume charges, or solid bodies nearby. With the aid of Figure 1.4-1 we can show that

$$V_p = \int_0^\pi \frac{\sigma 2\pi R^2 \sin\theta \, d\theta}{4\pi\varepsilon_o\sqrt{R^2 + d^2 - 2Rd\cos\theta}} = \frac{R^2\sigma}{\varepsilon_o d} = \frac{q}{4\pi\varepsilon_o d} \tag{1.4-15}$$

where R is the radius of the sphere, σ is the surface charge density, d is the distance from point P to the center of the sphere, and q is the total charge on the sphere.

Finally, let us note that, since the electric field at a given point is related to the potential gradient at the point by $\mathbf{E} = -\nabla V$ and since the gradient operator is linear, the total electric field is a vector sum of contributions arising from each of the separate point charges, and elements of surface charge and volume charge in the region. Hence superposition applies to fields as well as potentials.

1.5 Static Magnetic Fields — Ampere's Circuital Law, Permanent Magnets

Static magnetic fields always result from charge in motion — sometimes an electron current in a conducting medium, or a beam of charged particles, or, in the case of permanent magnets, a preferred orientation of the electron spins or orbits in the solid matter of which the magnets are made. As in the case of an electric field, it is often convenient to picture a magnetic field in terms of magnetic flux or magnetic field lines. The lines lie parallel to the direction of the magnetic flux density **B**, and the number of lines crossing unit area normal to the direction of the field is proportional to $|\mathbf{B}|$.

When current flows in a long cylindrical conductor and the direction of flow is parallel to the axis of the conductor, the magnetic field lines are circles concentric with the conductor and lying in a plane perpendicular to

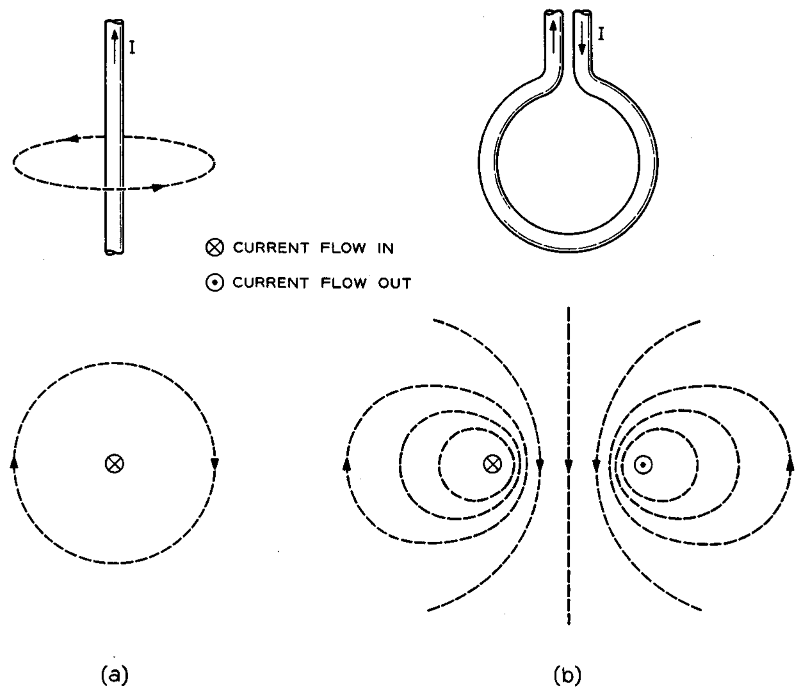


FIG. 1.5-1 Magnetic field lines associated with current flow in a wire and a loop.

the axis. The magnetic flux density is found to be greatest at the surface of the conductor and falls off inversely with distance from the axis at larger distances from the axis. Figure 1.5-1 illustrates the direction of the magnetic field in relation to the direction of current flow. If the conductor is bent in the form of a loop, the magnetic field lines still surround the flow of current, and each field line threads through the loop. In all cases the field lines close on themselves, and there are no point sources of magnetic field. Magnetic field lines never start or stop at a point or surface as do electric field lines.

Since the magnetic field lines close on themselves, the total magnetic flux crossing a closed surface must be zero. The magnetic flux crossing an element of area dS can be expressed as $\mathbf{B} \cdot \mathbf{n} dS$, where $\mathbf{n}$ is a unit vector normal to the element of area. Hence

$$\int_{\substack{\text{closed}\\\text{surface}}} \mathbf{B} \cdot \mathbf{n} dS = 0 \tag{1.5-1}$$

If the volume enclosed by the surface is very small and can be represented by Δv and if we take the limit as $\Delta v \rightarrow 0$, we obtain

$$\int_{\substack{\text{closed}\\\text{surface}\\\Delta v \rightarrow 0}} \frac{\mathbf{B} \cdot \mathbf{n} dS}{\Delta v} = \nabla \cdot \mathbf{B} = 0 \tag{1.5-2}$$

In the mks system the unit of magnetic flux is the weber, and magnetic flux density $\mathbf{B}$ is measured in webers per square meter.

For some purposes it is convenient to define a vector $\mathbf{H}$, known as the magnetic field intensity vector, such that

$$\mathbf{B} = \mu\mu_o\mathbf{H} \tag{1.5-3}$$

where μ is the relative permeability of the medium, and μ_o is the permeability of free space. The relative permeability μ is a dimensionless constant, which in free space is equal to 1. In mks units the constant μ_o is defined to be equal to $4\pi \times 10^{-7}$ and has the dimensions of henries per meter or webers per ampere-meter. Since $\mathbf{B}$ has the dimensions of webers per square meter, $\mathbf{H}$ has the dimensions of amperes per meter.

(*a*) *Ampere's Circuital Law*

Ampere's Circuital Law states that the line integral of $\mathbf{H}$ around any closed path which surrounds a flow of current I is equal to the flow of current across the area enclosed by the path, or

$$\oint_{\text{closed path}} \mathbf{H} \cdot d\mathbf{l} = I \tag{1.5-4}$$

If the closed path in this equation lies in a plane normal to a current density J and if the area surrounded by the closed path is very small and can be represented by ΔA, we can divide both sides of the equation by ΔA and take the limit as $\Delta A \to 0$ to obtain

$$\oint_{\substack{\text{closed path}\\ \Delta A \to 0}} \frac{\mathbf{H} \cdot d\mathbf{l}}{\Delta A} = J \tag{1.5-5}$$

or, since the left-hand side is the definition of the curl of **H**,

$$|\nabla \times \mathbf{H}| = J$$

and

$$\nabla \times \mathbf{H} = \mathbf{J}$$

where **J** is a vector parallel to the flow of current and of magnitude equal to J. Ampere's Circuital Law applies when the closed path lies within solid bodies, conductors, or magnetic materials, as well as in regions of free space.

Equation (1.5-4) can be used to obtain the magnetic field intensity at a distance a from the axis of a long cylindrical conductor in free space which conducts a current I amperes parallel to its axis. If the closed path in the equation is a circle of radius a and if the circle is normal to the axis of the conductor with center on the axis, so that H is parallel to the path at all points, we obtain

$$H 2\pi a = I \tag{1.5-6}$$

Hence the magnetic flux density B at a distance a from the axis of a long cylindrical conductor, which carries a current I and which is surrounded only by free space, is given by

$$B = \frac{\mu_o I}{2\pi a} \tag{1.5-7}$$

Actually the magnetic field generated by a long straight conductor is a vector sum of contributions resulting from each element of length of the conductor. Ampere deduced that when a current I amperes flows in an element of length dl of a conductor, the magnetic flux density $d\mathbf{B}$ at a point r meters from the length dl is given by

$$d\mathbf{B} = \frac{\mu_o I (d\mathbf{l} \times \mathbf{r})}{4\pi r^3} \tag{1.5-8}$$

where $d\mathbf{l}$ is a vector of length dl and direction parallel to the current flow. The vector $\mathbf{r}$ is of length r and directed away from the element dl along a line joining dl to the point at which $d\mathbf{B}$ is determined. This result is known as *Ampere's Rule*. It applies only when there is no magnetic material in the region. With the aid of Figure 1.5-2 it is easily shown that the sum of the contributions to the net magnetic flux density at a point a meters from the axis of a long cylindrical conductor which carries a current I amperes is given by

$$B = \int_{-\pi/2}^{\pi/2} \frac{\mu_o I \cos\phi \, d\phi}{4\pi a} = \frac{\mu_o I}{2\pi a} \qquad (1.5\text{-}9)$$

in agreement with Equation (1.5-7). Ampere's Rule is really a special form of the Circuital Law.

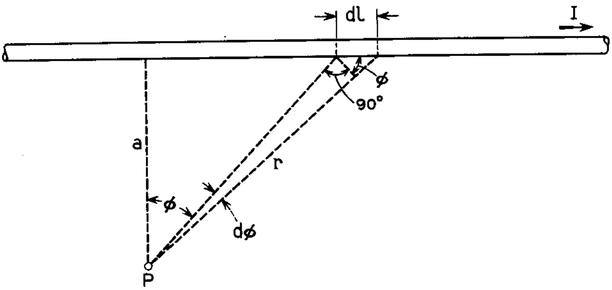


FIG. 1.5-2 A long cylindrical conductor carrying a current I amperes.

Equation (1.5-8) can in principle be used to determine the magnetic flux density at any point in space resulting from a coil of any shape, if sufficient ingenuity is used in carrying out the vector addition of the contributions $d\mathbf{B}$ from each element of current flow.

Perhaps the simplest application of Equation (1.5-8) is the problem of determining the magnetic flux density at the center of a circular loop of wire

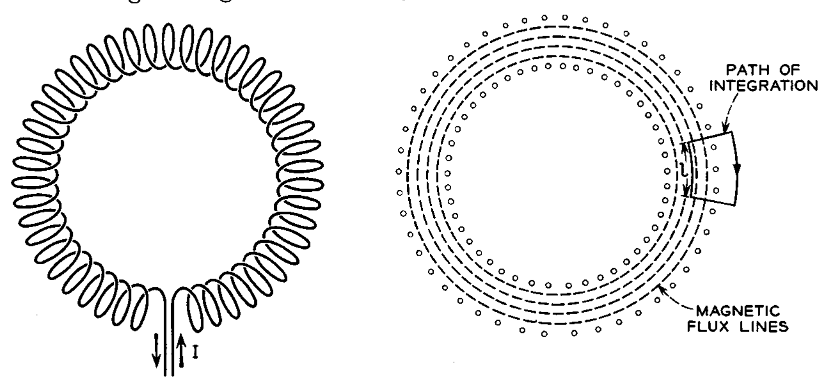


FIG. 1.5-3 Magnetic field lines associated with a toroidal coil which conducts a current I amperes.

which carries a current I and is of radius a. In this case the vectors $d\mathbf{B}$ at the center of the loop resulting from each element dl of the loop are all parallel. The total magnetic flux density at the center is easily shown to be

$$B = \frac{\mu_o I}{2a} \tag{1.5-10}$$

and is parallel to the axis of the loop.

Figure 1.5-3 shows qualitatively the shape of the magnetic flux lines associated with a toroidal coil. If the turns are close together and regularly spaced, it is evident from symmetry considerations that the magnetic field lines must all lie within the toroid and that B outside the coil is essentially zero. If there are n turns per unit length around the periphery of the coil, application of Ampere's Circuital Law to the path of integration shown in the figure gives

$$Hl = nlI$$

or

$$H = nI \tag{1.5-11}$$

where l is the length of the curved part of the path within the toroid. (The only non-zero contribution to the line integral comes from the curved part of the path within the toroid.) The magnetic flux density within the coil is therefore given by $B = \mu_o nI$. This is also the magnetic flux density at the center of a long straight coil of n turns per meter.

The inductance of a coil is equal to the number of "flux linkages" per ampere of current passed through the coil, where the number of flux linkages is equal to the product of the number of webers linking each turn of the coil and the number of turns in the coil. In the case of the toroidal coil shown in Figure (1.5-3), the flux linking each turn of the coil is $\pi r^2 B = \pi r^2 \mu_o nI$, where r is the radius of the turns. If the total number of turns in the coil is N, the number of flux linkages per ampere is $\pi r^2 \mu_o nN$, or

$$L = \pi r^2 \mu_o nN \tag{1.5-12}$$

where L is the inductance of the coil. In the mks system inductance is measured in henries. If the coil were filled with a medium of relative permeability μ, the inductance would be $L = \pi r^2 \mu \mu_o nN$.

(*b*) *Permanent Magnets*

A number of metals including the elements iron, nickel, and cobalt, and certain alloys, as well as a group of ceramics called ferrites, exhibit a property known as ferromagnetism. When a long cylindrical rod of one of these materials is placed along the axis of a coil and a current is passed through the coil, the magnetic flux density B within the rod is often hundreds or thousands of times that which would be obtained along the axis of the coil in the absence of the ferromagnetic material. The ratio of the magnetic flux density within the sample to that which would be obtained in free space with the same value of H is known as the relative permeability of the material and is designated by μ. The magnetic flux density B within the material can therefore be expressed as $B = \mu\mu_o H$, as in Equation (1.5-3).

Figure 1.5-4(a) shows a coil wound around a toroidal sample of ferro-

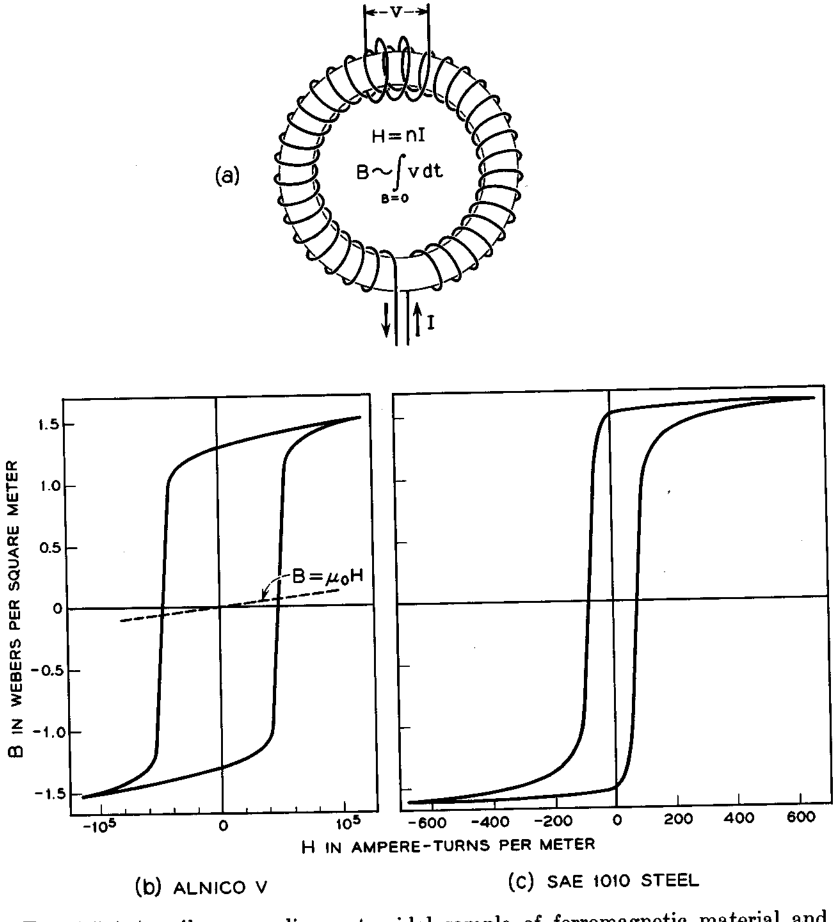


FIG. 1.5-4 A coil surrounding a toroidal sample of ferromagnetic material and hysteresis loops for two ferrous alloys. Alnico V is frequently used as a permanent magnet material, and SAE 1010 steel is often used for pole pieces.

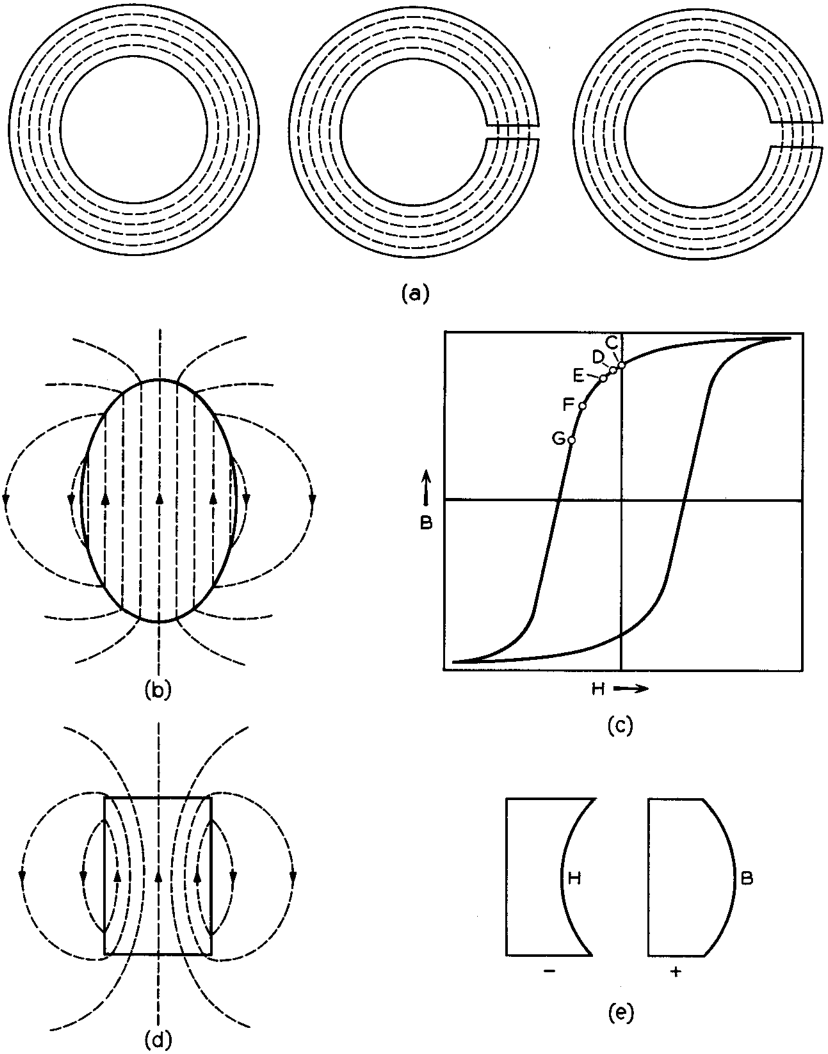


FIG. 1.5-5 The magnetic flux lines associated with several shapes of permanent magnets. (a) Three toroidal magnets, two with air gaps. A small amount of flux leakage which would take place from the sides of the two magnets with the air gaps is not shown. (b) An ellipsoidal magnet. (c) A hysteresis loop. (d) A cylindrical magnet. (e) Plots of B and H along the axis of the cylindrical magnet.

magnetic material. By passing a current I through the coil, a magnetic field intensity $H = nI$ is established within the sample, where n is the number of turns per unit length around the periphery of the toroid. If a low-frequency alternating current is passed through the coil, the magnetic flux density[3] B within the material is found to lag the applied H. The familiar "hysteresis loop" is a plot of B vs. H obtained in this manner. Two examples of hysteresis loops are shown in Figure 1.5-4. The shape of the hysteresis loop is characteristic of the particular ferromagnetic material. (Notice the difference in the horizontal scale for the two hysteresis loops shown in the figure.) Materials having hysteresis loops with large enclosed areas make the best permanent magnet materials.

Figure 1.5-5(a) shows three toroidal rings of ferromagnetic material. In one the ferromagnetic material forms a closed ring, in one there is a small air gap, and in one there is a larger air gap. We shall assume that each has been "magnetized" by winding a toroidal coil around it and momentarily passing a large current through the coil. When the magnetizing current is removed, the line integral of H around any closed path in the region must be zero, since there is no flow of current in or around the sample. From symmetry arguments we can easily deduce that within the closed ring, $H = 0$, and that B has the value indicated by point C on the hysteresis loop. The flux lines take the form of circles concentric with the axis of the toroid, and all are within the sample. There is no magnetic flux outside the sample.

In the case of the sample with the small air gap, nearly all the lines of flux cross the gap, so that B in the gap is approximately equal to B in the solid. However, since H is parallel to the direction of B in the gap and since the line integral of H along a path following the flux lines must be zero, H must be in the opposite direction to B in the magnetic material. It will be convenient to denote the values of B and H in the air gap with the subscript g and the values of B and H in the magnetic material with the subscript m. Then $B_g \approx B_m$. If H is integrated along a path followed by a flux line which crosses the center of the gap, we obtain

$$\oint \mathbf{H} \cdot d\mathbf{l} = lH_g + LH_m = 0 \tag{1.5-13}$$

where l is the length of the air gap, and L is the length of the path in the magnetic material. Evidently H_m is small and negative and the values of B_m and H_m might be those corresponding to point D on the hysteresis loop. Since B is positive, it follows from Equation (1.5-3) that μ for the magnetized toroid is negative. In the case of the sample with the larger air gap, the values of B_m and H_m corresponding to point E might apply. In both samples with the air gap there will actually be "flux leakage" outside the

[3]Changes in the magnetic flux density B within the sample are linearly proportional to the time integral of the voltage generated in an auxiliary coil surrounding the sample and can be measured in this manner.

gap since H is not zero within the sample, and $\oint \mathbf{H} \cdot d\mathbf{l}$ must equal zero for all closed paths.

Figure 1.5-5(b) shows qualitatively the shape of the magnetic flux lines associated with an ellipsoidal sample of ferromagnetic material when the sample is magnetized parallel to the long axis of the ellipsoid. It can be shown that, when an ellipsoidal sample is magnetized parallel to one of its axes, the B lines within the sample are all parallel to each other and to the axis. The values of B_m and H_m in this case might correspond to point F on the hysteresis loop.

Figure 1.5-5(d) shows qualitatively the shape of the field lines associated with a cylindrical bar magnet[4]. Some of the flux lines leave the sample through the sides in this case, with the result that B is less at the ends than at the center. Consequently, although the values of B_m and H_m at the center of the magnet might correspond to point F on the hysteresis loop, the values of B_m and H_m at the ends might correspond to point G. Figure 1.5-5(e) shows qualitatively the variation of H and B along the axis of the bar magnet.

From the foregoing discussion it is apparent that the operating point on the hysteresis loop is determined by the geometry of the permanent magnet. To illustrate this point further, let us return to the two toroidal magnets with air gaps illustrated in Figure 1.5-5(a). If it is assumed that all the lines of B cross the gap and that there is no flux leakage from the sides of the magnet, then

$$B_m = B_g = \mu_o H_g \qquad (1.5\text{-}14)$$

Combining this with Equation (1.5-13), we obtain

$$\frac{B_m}{H_m} = -\frac{\mu_o L}{l} \qquad (1.5\text{-}15)$$

This defines the slope of a line through the origin of the coordinate system for the hysteresis loop, and the intersection of this line with the hysteresis loop defines the operating point for B_m and H_m.

Since $\oint \mathbf{H} \cdot d\mathbf{l} = 0$ for all closed paths in the neighborhood of a permanent magnet, it is possible to define a magnetic potential ψ such that the potential difference between points A and B is given by $\psi_{AB} = -\int_A^B \mathbf{H} \cdot d\mathbf{l}$. (The magnetic potential difference between two points is often called the magnetomotive force, or mmf, in analogy to the electromotive force, or emf, in electrostatics.) The magnetic field intensity is related to the magnetic potential ψ by $H = -\nabla\psi$. Since $\mathbf{B} = \mu_o \mathbf{H}$ in the region *outside* a per-

[4]After M. Abraham, R. Becker, *Classical Theory of Electricity and Magnetism*, p. 137, Blackie and Son, 1932.

manent magnet, and since $\nabla \cdot \mathbf{B} = 0$ and $\nabla \cdot (\nabla) = \nabla^2$, the magnetic potential in the space surrounding a permanent magnet satisfies Laplace's Equation, $\nabla^2 \psi = 0$.

Magnetic fields are used to focus, or confine, the electron beams of a number of microwave tubes including traveling-wave tubes, klystron amplifiers, and backward-wave oscillators. Magnetic fields also play an

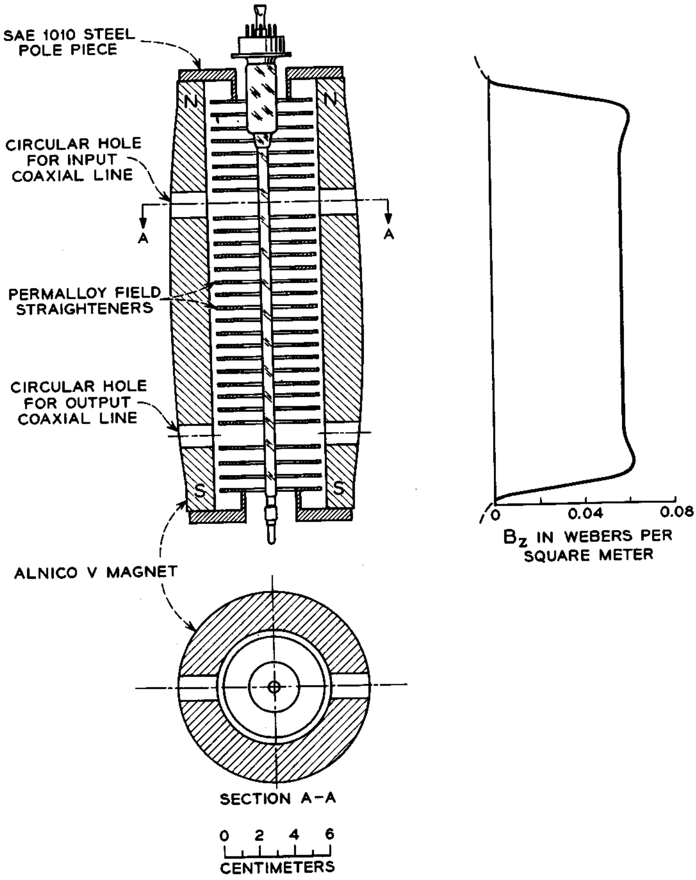


FIG. 1.5-6 A permanent magnet circuit used to focus the electron beam of a traveling-wave tube. The outline of the tube is shown in the figure. A plot of the axial magnetic field B_z is shown at the right. The slight peaking of the axial magnetic field near the ends of the circuit results from the "re-entrancies" in the pole pieces. Within the pole pieces the axial magnetic field changes direction, and beyond the pole pieces the axial magnetic field has the opposite direction to that which it has in the center of the magnet.

essential role in the operation of magnetron oscillators. By using permanent magnets rather than electromagnets to provide the magnetic field, the total power consumption of the tubes can be reduced.

Figure 1.5-6 shows a permanent magnet circuit for a traveling-wave tube. The circuit produces a magnetic flux density[5] of nearly 0.06 weber/meter^2 along the axis of the tube in the region between the pole pieces. The magnetic flux density B in the pole pieces is well below that needed to saturate the pole piece material, so that H within the pole pieces is extremely small (see hysteresis loop for SAE 1010 steel in Figure 1.5-4). The pole pieces, therefore, serve as equipotential bodies, the mmf being nearly constant throughout their volume. In a similar manner, the permalloy "field straighteners" are flat discs of high-permeability steel which serve as equipotential planes and assure that the lines of B are parallel to the axis of the traveling-wave tube. Since $B = \mu\mu_o H$, and μ is very large for the field-straightener material, H within the field straighteners is correspondingly small. The permanent magnet is larger at its center than at its ends to account for flux leakage from its sides.

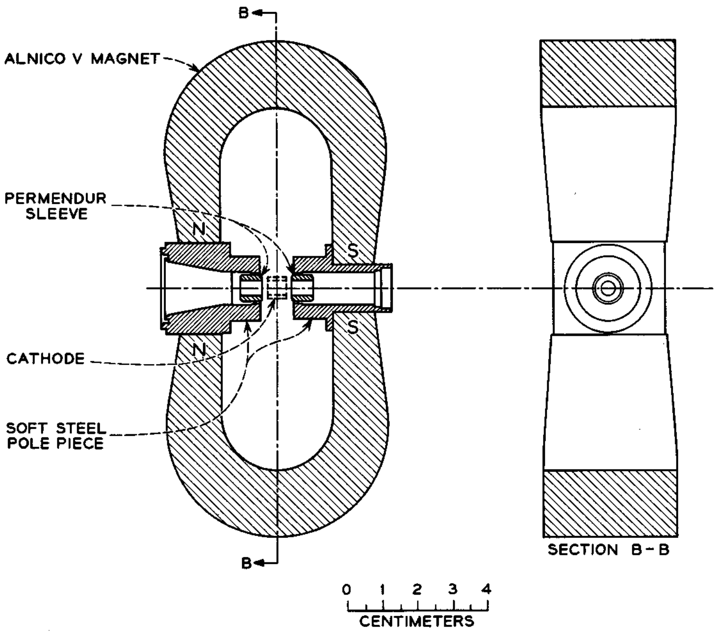
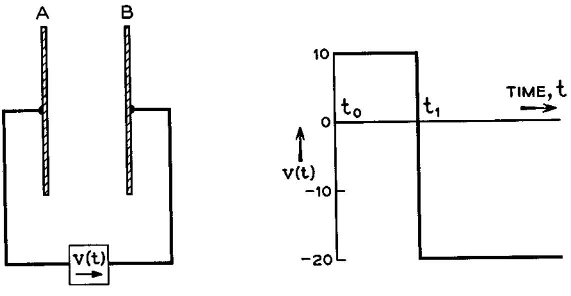


Fig. 1.5-7 A permanent magnet circuit for a magnetron.

[5]One weber per square meter = 10^4 gauss.

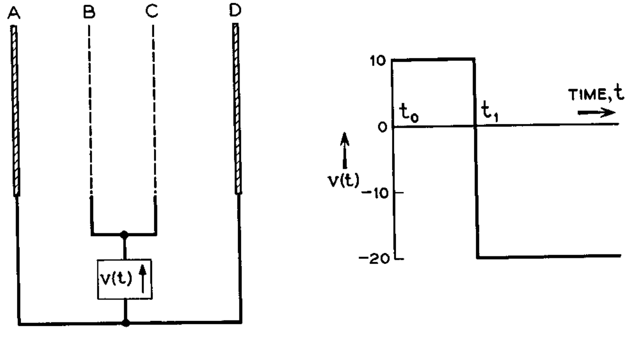
Figure 1.5-7 shows a permanent magnet circuit for a magnetron. The circuit produces a magnetic flux density of about 0.5 weber/meter^2 in the neighborhood of the magnetron's cathode. The permendur sleeves inside the pole pieces serve to shape the magnetic field in the region between the cathode and anode so as to obtain electron trajectories which give optimum interaction between the electrons and the rf field.

PROBLEMS



Problem 1.1

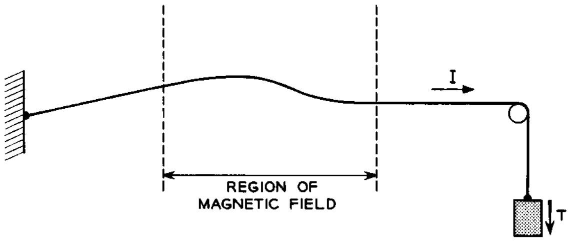
1.1 At time t_o a single electron is emitted from electrode A with zero velocity, and at this time a voltage $V = +10$ volts is applied between the electrodes in such a direction that it accelerates the electron toward electrode B. It is assumed that the electric field intensity is uniform at all points between the electrodes. At time t_1 the electron is halfway to electrode B, and the voltage V changes discontinuously to -20 volts and remains at that value. Which electrode does the electron strike, and what is its kinetic energy in electron volts when it strikes the electrode?



Problem 1.2

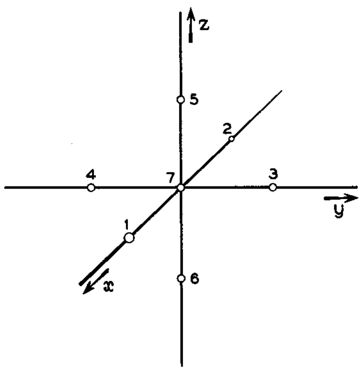
1.2 Grids B and C are assumed to be ideal grids having the properties that they do not intercept electrons and that field lines do not penetrate through the grids.

A single electron leaves electrode A with zero velocity at time t_o. At this time the voltage V is $+10$ volts and is in such a direction that the electron is accelerated toward grid B. At time t_1 the electron is midway between grids B and C, and the voltage V changes to -20 volts. Which electrode (either D or A) does the electron strike, and what is its kinetic energy in electron volts when it strikes the electrode?



Problem 1.3

1.3 A very fine wire is held stationary at one end, while the other end passes over a pulley and is fixed to a weight which maintains a tension T newtons in the wire. Over a limited region between the fixed end of the wire and the pulley there is a magnetic field that varies across the region both in magnitude and direction. If a current I amperes is passed through the wire, the magnetic field causes a force to act on the wire which tends to deflect it. The force is equal to BI newtons per meter length of the wire and acts in the direction normal to both the current flow and the magnetic field. The resulting shape of the wire might be that shown in the figure. Suppose that the wire were removed and that an electron were directed toward the magnetic field along the path previously followed by the wire. Show that, if the electron momentum mu satisfies the relation $mu/e = T/I$, the electron trajectory through the region of the field will coincide with the path followed by the wire. Assume that the stiffness of the wire can be neglected and that its mass is negligible.



Problem 1.4

1.4 Points 1, 7, and 2 lie on the x axis of a rectangular coordinate system. Points 3, 7, and 4 lie on the y axis, and points 5, 7, and 6 lie on the z axis. The distance from point 7 to each of its neighboring points is d meters. The region is filled with a uniform charge density ρ_o coulombs/meter^3. Show that if the distance d is very small, the potential at point 7 is approximately given by

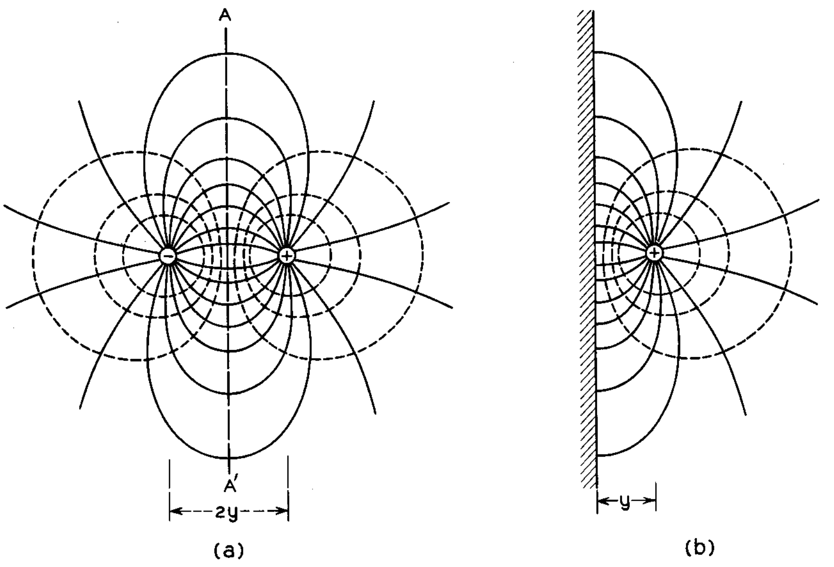
$$V_7 = \frac{V_1 + V_2 + V_3 + V_4 + V_5 + V_6}{6} + \frac{d^2\rho_o}{6\varepsilon_o}$$

where V_1 is the potential at point 1, and so on. What effect does the presence of space charge have on the potential at point 7?

1.5 Use Equation (1.4-14) to show that the potential at the center of an isolated spherical cloud of charge of radius R and uniform charge density ρ_o is given by

$$V_{\text{center}} = \frac{\rho_o R^2}{2\varepsilon_o} = \frac{3q}{8\pi\varepsilon_o R}$$

where q is the total amount of charge in the cloud.



Problem 1.6

1.6 Part (a) of the figure shows qualitatively the field lines associated with two isolated point charges $+q$ and $-q$. The plane A-A' lies midway between the two point charges. Since all points on the plane are equidistant from the two point charges, the potential on the plane is zero. Both charges contribute to the electric field intensity at the plane A-A'. Show that the total electric field intensity at the plane can be expressed as

$$E = \frac{qy}{2\pi\varepsilon_o(r^2 + y^2)^{3/2}}$$

where y is the distance from the point charges to the plane A-A', and r measures the distance along the surface of the plane from the line joining the point charges to the point at which E is determined. The electric field intensity at the plane A-A' is, of course, normal to the plane.

Since all points on the plane A-A' are at zero potential, a thin planar conductor could be inserted along the plane without disturbing the potential and field distribution in the region. Suppose such a planar conductor were inserted and the left-hand charge were then removed. Evidently the right-hand half of the field pattern would remain unchanged. Hence the field distribution shown in part (b) of the figure must be that which applies when a point charge $+q$ is y meters from planar conductor. Field lines originating on the charge $+q$ terminate on negative induced charges on the surface of the conductor. Use the above expression for E to obtain an expression for the surface charge density induced on the planar conductor by the charge $+q$. Show that the total induced charge is equal to $-q$.

Show that the force tending to draw the charge $+q$ toward the planar conductor in part (b) of the figure is $q^2/[4\pi\varepsilon_o(2y)^2]$ newtons and that the work required to remove the charge $+q$ to infinite distance from the planar conductor is $q^2/[4\pi\varepsilon_o(4y)]$ joules.

1.7 A dc current I amperes flows within a long cylindrical conductor of radius R. The current density is assumed to be uniform across the wire and directed parallel to the axis. Sketch qualitatively how the magnetic flux density B varies with radius r from the axis of the wire out to several times R. Make a similar sketch for the radial electric field intensity associated with a cylindrical beam of electrons. Assume uniform space charge density across the beam cross section.

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